Generalized Discrete-Time Models for Descriptor Systems with Arbitrary Initial Conditions

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Generalized Discrete-Time Models for Descriptor Systems with Arbitrary Initial Conditions

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Abstract

The present thesis is concerned mainly with the generalized mapping discrete-time models of a regular, linear, and time-invariant, descriptor system, whose initial condition can be selected in an appropriate manner corresponding to an initial condition given arbitrarily to the continuous-time original. Since a continuous-time descriptor system can involve impulsive responses, the concept of discrete-time distribution is introduced and presented in the form of definition, which takes values at multiple discrete-time instants into account, in contrast to a single instant that is sufficient to ordinary functions. By applying this idea to the input and the output, the discretization of a continuous-time descriptor system that generates impulsive responses can be handled without ambiguity.

Besides these main results, two step-aside topics are included. One is the derivation of a formula to select a suitable initial condition for the state-space discrete-time model of linear time-invariant continuous-time systems; the two important examples being the exact and the mapping discrete-time models. The other is a number of new insights into a system order; it is pointed out that a change in the order can be interpreted as a shift of a mode among the exponential, static, and impulsive ones of a descriptor system.

1 Introduction

1.1 Discretization and Descriptor Systems

The use of digital devices for analysis and design of a wide variety of systems and controllers has become a common practice lately. Physical laws, such as Newton's law of motion, are available in continuous-time form but not in discrete-time form. Thus, a system must first be modeled in an analog format. Therefore, to implement algorithms on digital control and simulations, a system expressed by discrete-time signals that can be handled by digital device is necessary. Thus, it is desirable to develop a discretization method as a method to convert a continuous-time system into a discrete-time system that preserves various characteristics such as the function and performance that it is desired to inspect. While a large number of methods exist for discretizing a continuous-time system that appears in many fields of modern science and engineering, the concept of discretization is not always clearly defined in the context of digital control [1]–[4]. In [5], one such definition of discretization was proposed in terms of a point-wise closeness between signals of different domains; continuous-time and discrete-time. This definition has paved a way to developing a series of useful theorems for linear time-invariant systems [6] and linear time-variant systems [7].

More recently, based on these insights, new discretization methods were proposed for a number of nonlinear systems [8], [9], using such techniques as linearizations [10], [11] and continualizations [12]–[14].

These definitions, theorems, and methods of discretization have been proposed mostly for systems expressed in the transfer function and state-space forms, while there has been few for the descriptor system [15], which is one of the mathematical models and also called the generalized state-space form [16]. As systems become increasingly more complex, the latter model with more flexibilities than the former two forms are increasingly more appreciated in dealing with various phenomena observed in practice, such areas as large-scale systems [17], [18], singularly perturbed systems [19], [20], noncausal system such as differentiators [21], switched systems [16], [22], [23], and inverse systems [18], [24]. These systems often involve subsystems that cannot be handled properly by state-space equations and require the descriptor system expression. Their fundamental properties are investigated in [25] and control applications have been discussed in [26]–[28]. Moreover, their numerical tools can be found in [29], [30]. If a discrete-time descriptor system is used to reproduce a continuous signal numerically, a computation must be started from a certain condition at a convenient instant, called an initial condition. One of the long-standing issues in numerical computations for descriptor forms is that the discrete-time signal can, and usually does, show an impulsive response at the initial time, even though there is none in the continuous-time signal. In other words, spurious impulses show up at the instant a computation is started. This is inconvenient and has to be solved completely, which will be achieved in this thesis.

Another aspect of the descriptor system considered in this thesis is order changes. Such changes cannot be handled easily using the state-space form [31], [32], but can be done using the descriptor form. This proves convenient since the order of a system is often determined by trials and errors. Real systems have infinite orders but usually are approximated as finite ones for easier handling. However, there seems to have been no clear-cut criteria for determining the order of a system. Since the state-space form assumes a fixed order, every time the order is changed, the dimension and usually the parameter values appearing in the state-space form have to be modified. In particular, without knowing how the parameter values are affected by the order change, the effects of order changes to controller parameters are unpredictable; a change in a plant parameter might cause changes in all the controller parameters in discontinuous manners. This can be alleviated greatly using the transfer function in the anti-monic form [33], and its equivalent form in the descriptor form is desired. This will be accomplished in this thesis.

The denominator polynomial of a transfer function is often made to be monic, which is to normalize the polynomial so that the coefficient of the highest order term is unity. This enables one to realize the transfer function in a state-space expression. However, making the characteristic equation monic is essentially equivalent to declaring that the system order is fixed to a certain number and, thus, an order change cannot be accommodated thereafter. By making the characteristic non-monic and allowing the highest-order coefficient to approach zero, the order can be reduced by one. In reverse, the order can be increased by adding a higher order term with a nonzero coefficient. In the same vein, it was pointed out in [33] that the anti-monic expression of transfer functions provides a clear perspective view when order changes are to be taken into account in modeling systems and designing controllers. As in the state-space form that can provide internal descriptions of transfer functions and matrices, the descriptor form turns out to be highly valuable to explain the mechanics of various modes and interactions among them.

For numerical analyses and subsequent designs of digital controllers, a continuous-time descriptor expression needs to be discretized. Therefore, a number of studies have been carried out on the topic of discretization for descriptor systems from the numerical analysis point of view [34]–[37]. For instance, the forward-difference approximation of the differentiator is used to discretize the impulsive mode of the descriptor system in [34] and shown to be equivalent to the model obtained by applying the forward-difference approximation to the derivative of the input that appears in the analytical solution of the descriptor system. The resulting model is non-causal and cannot be used for online computations. Markov parameters are used in [35] to discretize the descriptor system, where again the forward-difference approximation is applied to the derivative terms of the input. This resulted in the same model as one obtained in [34]. Discretization from the viewpoint of a solution to the system is attempted in [36], where the derivative of the input term is approximated by the backward-difference. This model is identical to one proposed in [37], where the impulsive mode is discretized using the backward-difference. Unlike the forward-difference model [34], [35], the backward-difference model [36], [37] is causal and can be used for on-line computations. While the relationship among these studies is not clear in their respective form, the model reported in [38] combines them in a unified framework and includes them as its special cases, giving a perspective view and suggesting new models. It is also pointed out in [38] that Tustin's method (bi-linear discretization), which is commonly known to produce good results for state-space expressions, leads to a marginally stable discrete-time model, even though the continuous-time original is stable.

There are possibly three modes that exist in a continuous-time descriptor system. The first is the exponential mode, which has a finite eigenvalue, or natural frequency, and its response is exponential. This mode can be modeled by a state-space equation. The second is the static mode, which has an infinite eigenvalue and the output is a scaled version of the input. The response will show a jump if the input has a jump. The third is the impulsive mode, which also has an infinite eigenvalue and the output is a differentiated (possibly multiple times) version of the input. The response can involve an impulse and its higher-order derivatives. The static and impulsive modes are the ones that cannot be expressed in an ordinary state-space equation. These modes of a descriptor system are necessary to model response behaviors at an instant, usually chosen to be an initial time, and can exhibit jumps, impulses, and their derivatives there. In continuous-time descriptor systems, such discontinuous responses can occur depending on the initial condition of the system and the type of input applied to it [25]. A point to note is that initial conditions and inputs should be considered as a set in calculating responses of a descriptor system, since the response to the initial condition and that to an input are joined by an impulsive input. That is, the impulse can be considered as an input as well as a tool to set up an initial condition. Therefore, the input has to be chosen to satisfy a certain condition to have an impulsive response (a consistent initial condition, C-IC). When this condition is not satisfied, there will be impulses (an inconsistent initial condition, IC-IC). To be more specific, the consistent initial condition is a class of initial conditions on static and impulsive modes of descriptor systems, which is necessary and sufficient for the response to be continuous at the initial time.

1.2 Issues to Tackle

Rather surprisingly, there seems to have been no clear solution presented on the choice of proper initial conditions for a discrete-time descriptor system when the initial condition to the continuous-time descriptor system are given arbitrarily. Therefore, until this thesis, a discrete-time descriptor system could exhibit unexpected impulsive responses when no such responses occur in the continuous-time system, and vice versa. Such discrepancies must be resolved for a discrete-time model to be useful for analyses, simulations, and designs of, a descriptor system that involves static and impulsive modes. This will be achieved in this thesis.

Since a continuous-time system expressed in a descriptor form can exhibit impulsive responses [39], discretization of Dirac's delta function and its derivatives has to be defined. While discretization has long been researched in such fields as numerical analysis, signal processing, and digital control, no such attempt has been successful for generalized functions, called distributions, as far as the author of this thesis is aware. Ordinary functions, which do not involve infinity, have been tackled successfully in [5] under the constraints of online computability. This uses a closeness measure between a continuous-time and discrete-time functions based on instantaneous values. By extending this idea to one based on values at multiple time-instants using an inner-product, the concept of discretization can be clearly defined, as shown in this thesis.

The main and final goal of the present thesis is, therefore, to

> "develop a discrete-time model of a descriptor system with an appropriate initial condition, given an arbitrary condition to the continuous-time original."

As a necessary step to achieve this goal,

> "a definition is proposed for discretization of an impulse and its derivatives, and useful theorems developed based on this definition."

As a sort of by-product of the research,

> "an order change is shown to be interpretable as a shift of a mode among the exponential, static, and impulsive ones."

1.3 Organization of the Thesis

The main body of the thesis is organized as follows:

Chapter 1: Introduction
Brief reviews of topics that are pertinent to the thesis are provided.

Chapter 2: Preliminaries
This chapter reviews existing definitions and theorems on discretization, descriptor systems, and distributions, which appear in later chapters.

Chapter 3: Order-Changes in Terms of Mode-Shifts
Considers the advantage of a descriptor form over state-space and transfer-function expressions. [C2]

Chapter 4: Discrete-Time Model of a State-Space System with Initial Conditions
A proper choice of initial conditions for the mapping discrete-time model of a continuous-time state-space system is presented explicitly, which has been unclear in the past. [C1]

Chapter 5: Discrete-Time Model of a Descriptor System with Consistent Initial Conditions
A generalized mapping discrete-time model is proposed where the initial conditions are assumed to be consistent. This is actually a class of models with a design parameter varied. The stability of the discrete-time model is investigated and the range of the design parameter derived. [J1]

Chapter 6: Generalized Discretization of Continuous-Time Distributions

A definition of discretization of functions is extended to distributions, by evaluating a closeness of discrete and continuous-time signals at multiple time-instants, rather than a single instant as in a conventional definition. [Paper in preparation]

Chapter 7: Discrete-Time Model of a Descriptor System with Arbitrary Initial Conditions

This chapter presents a formula for properly determining the initial condition of the discrete-time model of a continuous-time descriptor system, given any input signals. This formula requires values at multiple sampling points and cannot be obtained directly from the initial condition for the continuous-time case. With this choice, both consistent and inconsistent initial conditions can be covered in a unified manner. [Paper in preparation]

Chapter 8: Conclusions

This chapter presents conclusions of the thesis.

1.4 Contributions of the Thesis

The thesis claims the following as its original contributions: The thesis

1. gives a unique explanation of what happens inside a descriptor system when its order changes, in terms of shifts among the exponential, static, and impulsive modes.
2. proposes a general mapping model as a valid discrete-time model of a continuous-time descriptor system. This includes the existing forward and backward-difference models as its special cases and gives a class of valid models by changing a design parameter. The range of this parameter is investigated for assuring the stability of the proposed model.
3. derives a formula for determining a proper initial condition of the proposed discrete-time descriptor system, given an initial condition of the continuous-time descriptor system and the input to the system. This gives a consistent, discrete-time, initial condition when the continuous-time original is consistent, while it gives an inconsistent one when the original is inconsistent. Therefore, arbitrary conditions can be handled, for the first time.

2 Conclusions

The present thesis has dealt with the discretization of continuous-time, linear, time-invariant systems expressed in the generalized state-space form, called the descriptor system. In point form, the following topics are covered:

1. Proposal of a new definition of discretization that is applicable to generalized functions, or distributions, such as Dirac's delta function and its derivatives, while still accommodating ordinary functions without modifications.
2. Development of theorems based on the above definition, one of which give sufficient conditions for discrete-time descriptor systems to be valid discrete-time models in the sense of the above definition. Others include some convenient tools in handling the models, which lead to the next item.
3. Formulation of a general discrete-time model as a valid discrete-time model of a continuous-time descriptor system. This gives a perspective view of existing models by combining them into a single framework of a generalized mapping model. It includes the existing forward and backward-difference models as its special cases. Furthermore, it gives a class of valid models by changing a design parameter. The range of this parameter to assure the stability of the proposed model is also clarified.
4. Suggestion of a new interpretation of impulsive and static modes in a descriptor system from the order-change point of view. A new explain of what happens inside a descriptor system when its order changes, in terms of shifts of modes among the exponential, static, and impulsive modes, is also presented.
5. Resolution of the longstanding issue on the choice of initial conditions for the mapping discrete-time descriptor system to be a valid discrete-time model. A formula for determining a proper initial condition of the proposed discrete-time descriptor system, given an initial condition of the continuous-time descriptor system and the value of the input to the system. This gives a consistent, discrete-time, initial condition when the continuous-time original is consistent, while it gives an inconsistent one when the original is inconsistent. Therefore, arbitrary conditions can be handled, for the first time.

With these results, the main goal of the present thesis listed at the beginning of the work has been fulfilled; that is, the thesis has developed a discrete-time model of a descriptor system with an appropriate initial condition, given an arbitrary condition to the continuous-time original, by proposing a definition for discretization of an impulse and its derivatives, and useful theorems developed based on this definition. As a by-product, an order change has been shown to be interpretable as a shift of a mode among the exponential, static, and impulsive ones.

References

[1] T. T. Hartley, G. O. Beale, and S. P. Chicatelli, *Digital simulation of dynamic systems: a control theory approach*, ser. Electrical Engineering: Control Theory. PTR Prentice Hall, 1994.

[2] J. D. Lambert, *Computational methods in ordinary differential equations*, ser. Introductory mathematics for scientists and engineers. Wiley, 1973.

[3] R. H. Middleton and G. C. Goodwin, *Digital Control And Estimation — A Unified Approach*. New Jersey: Prentice Hall, 1990, p. 538.

[4] R. E. Mickens, *Nonstandard Finite Difference Models of Differential Equations*. 1993.

[5] T. Mori, P. N. Nikiforuk, M. Gupta, and N. Hori, "A class of discrete-time models for a continuous-time system," *IEE Proceedings D Control Theory and Applications*, vol. 136, no. 2, pp. 79–83, 1989.

[6] N. Hori, T. Mori, and P. N. Nikiforuk, "A new perspective for discrete-time models of a continuous-time system," *IEEE Transactions on Automatic Control*, vol. 37, no. 7, pp. 1013–1017, Jul. 1992.

[7] H. Shiobara and N. Hori, "Numerical exact discrete-time-model of linear time-varying systems," in *2008 International Conference on Control, Automation and Systems*, IEEE, Oct. 2008, pp. 2314–2318.

[8] N. Hori and C. A. Rabbath, "Application of digital control theory to exact discretization of a logistic equation with a constant term," in *Proceedings of 2005 IEEE Conference on Control Applications, 2005. CCA 2005.*, IEEE, 2005, pp. 303–307.

[9] N. Hori, C. A. Rabbath, and P. N. Nikiforuk, "Exact discretisation of a scalar differential Riccati equation with constant parameters," *IET Control Theory & Applications*, vol. 1, no. 5, pp. 1219–1223, Sep. 2007.

[10] Y. Tomita and N. Hori, "Exact discrete-time models for a class of second-order nonlinear systems," *33rd SICE Symposium on Intelligent Systems*, pp. 127–130, 2006.

[11] T. Sakamoto, N. Hori, and Y. Ochi, "Exact linearization and discretization of nonlinear systems satisfying a lagrange pde condition," *Transactions of the Canadian Society for Mechanical Engineering*, vol. 35, no. 2, pp. 215–228, 2011.

[12] T. Nguyen-Van and N. Hori, "Discretization of Nonautonomous Nonlinear Systems Based on Continualization of an Exact Discrete-Time Model," *Journal of Dynamic Systems, Measurement, and Control*, vol. 136, no. 2, p. 021 004, Nov. 2013.

[13] T. Nguyen-Van, N. Hori, and M. Nahon, “A discrete-time model of nonlinear non-autonomous systems,” in *2014 American Control Conference*, IEEE, Jun. 2014, pp. 5150–5155.

[14] T. Nguyen-Van and N. Hori, “Riccati-Based Discretization for Nonlinear Continuous-Time Systems,” *Journal of Computational and Nonlinear Dynamics*, vol. 11, no. 5, Feb. 2016.

[15] D. Luenberger, “Dynamic equations in descriptor form,” *IEEE Transactions on Automatic Control*, vol. 22, no. 3, pp. 312–321, Jun. 1977.

[16] G. C. Verghese, B. Levy, and T. Kailath, “A generalized state-space for singular systems,” *IEEE Transactions on Automatic Control*, vol. 26, no. 4, pp. 811–831, Aug. 1981.

[17] H. H. ROSENBROCK and A. C. PUGH, “Contributions to a hierarchical theory of systems,” *International Journal of Control*, vol. 19, no. 5, pp. 845–867, May 1974.

[18] T. Shiotsuki, *Analysis of linear systems*. Corona Publishing, 2011, p. 240, (in Japanese).

[19] F. L. Lewis, “A survey of linear singular systems,” *Circuits, Systems, and Signal Processing*, vol. 5, no. I, pp. 3–36, Mar. 1986.

[20] M. Suzuki, Y. Hayakawa, K. Yasuda, and S. Hosoe, *Dynamical system theory*. Corona Publishing, 2000, (in Japanese).

[21] T. Katayama, *Optimal control of linear systems — introduction to descriptor systems*. Kindaikagaku, 1999, (in Japanese).

[22] S. L. Campbell, *Singular systems of differential equations I*. London: Pitman Publishing, 1980, vol. 1, p. 176.

[23] A. Geerts and J. M. Schumacher, “Impulsive-smooth behavior in multimode systems part I: State-space and polynomial representations,” *Automatica*, vol. 32, no. 5, pp. 747–758, May 1996.

[24] K. Yamada, “Design method of inverse systems and their application,” *Journal of the Society of Instrument and Control Engineers*, vol. 36, no. 6, pp. 417–425, 1997, (in Japanese).

[25] E. Yip and R. Sincovec, “Solvability, controllability, and observability of continuous descriptor systems,” *IEEE Transactions on Automatic Control*, vol. 26, no. 3, pp. 702–707, Jun. 1981.

[26] J. D. Cobb, “Descriptor variable systems and optimal state regulation,” *IEEE Transactions on Automatic Control*, vol. 28, no. 5, pp. 601–611, May 1983.

[27] D. Bender and A. Laub, “The linear-quadratic optimal regulator for descriptor systems,” *IEEE Transactions on Automatic Control*, vol. 32, no. 8, pp. 672–688, Aug. 1987.

[28] K. Takaba and T. Katayama, “H2 Output Feedback Control for Descriptor Systems,” *Automatica*, vol. 34, no. 7, pp. 841–850, Jul. 1998.

[29] A. Varga, "A Descriptor Systems Toolbox for MATLAB," *CACSD Conference Proceedings IEEE International Symposium on ComputerAided Control System Design Cat No00TH8537*, pp. 150–155, 2000.

[30] A. Vardulakis, N. P. Karampetakis, E. Antoniou, and S. Vologiannidis, "Descriptor systems toolbox : a Mathematica-based package for descriptor systems," in *2008 IEEE International Conference on Computer-Aided Control Systems*, IEEE, 2008, pp. 595–600.

[31] T. Shiotsuki, "Introduction to implicit system model," *Journal of The Society of Instrument and Control Engineers*, vol. 36, no. 7, pp. 496–502, 1997, (in Japanese).

[32] ——, "Modeling and analysis by using implicit system model," *IEICE technical report. Reliability*, vol. 97, no. 351, pp. 1–6, Oct. 1997, (in Japanese).

[33] T. Kitamori, "Principle, synthesis and design of I-PD control," *Systems, control and information*, vol. 42, no. 1, pp. 7–17, 1998, (in Japanese).

[34] A. Rachid, "A remark on the discretization of singular systems," *Automatica*, vol. 31, no. 2, pp. 347–348, 1995.

[35] N. P. Karampetakis, "On the discretization of singular systems," *IMA Journal of Mathematical Control and Information*, vol. 21, no. 2, pp. 223–242, Jun. 2004.

[36] A. D. Karageorgos, A. a. Pantelous, and G. I. Kalogeropoulos, "Discretizing LTI Descriptor (Regular) Differential Input Systems with Consistent Initial Conditions," *Advances in Decision Sciences*, vol. 2010, pp. 1–19, 2010.

[37] R. Sincovec, A. Erisman, E. Yip, and M. Epton, "Analysis of descriptor systems using numerical algorithms," *IEEE Transactions on Automatic Control*, vol. 26, no. 1, pp. 139–147, Feb. 1981.

[38] S. Kawai and N. Hori, "On bi-proper mapping discretization of a regular descriptor system," *IECON Proceedings (Industrial Electronics Conference)*, no. 3, pp. 2366–2371, 2012.

[39] Z. Yan and G.-R. Duan, "Time domain solution to descriptor variable systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1796–1799, Nov. 2005.